## Subgraph And Spanning of Subgraph

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MSCS 3B

## Introduction

$\square$ What is a graph $G$ ?

- It is a pair $G=(V, E)$, where
- $\mathrm{V}=\mathrm{V}(\mathrm{G})=$ set of vertices
- $E=E(G)=$ set of edges
- Example:
- $V=\{s, u, v, w, x, y, z\}$
- $E=\{(x, s),(x, v),(x, v),(x, u)$, (v,w), (s,v), (s,u), (s,w), (s,y), (w,y), (u,y), (u,z),(y,z)\}



## Sub Graph

$\square$ A graph whose vertices and edges are subsets of another graph.
$\square$ A subgraph $\mathrm{G}^{\prime}=\left(\mathrm{V}^{\prime}, \mathrm{E}^{\prime}\right)$ of a graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ such that $V^{\prime} \subseteq V$ and $E \prime \subseteq E$, Then $G$ is a supergraph for G'.

(G)


## Subgraphs

E Example: $\boldsymbol{H}_{l}, \boldsymbol{H}_{2}$, and $\boldsymbol{H}_{3}$ are subgraphs of $\boldsymbol{G}$

..Sub Graph

(G1)

## Spanning Subgraph

$\square$ A spanning subgraph is a subgraph that contains all the vertices of the original graph.


## Spanning subgraphs

- Given a grapg $G=(V, E)$, let $G_{1}=\left(V_{1}, \mathrm{E}_{1}\right)$ be a subgraph of $G$. ilf $V_{1}=V$ then $G_{1}$ is called a spanning subgraph of $G$.



## Real-World Applications of Graphs and Graph Theory

- Modeling a network of computer systems
- Modeling an online social network
- Modeling a physical social network
- Modeling airline routes
- Modeling road networks
- Modeling recursive functions (recall?)
... Many other applications (refer to the textbook)
In this course, we focus more on the properties of abstract graphs rather on graph algorithms


## Computer network

- graph theory used in all types of topology for configure network.
- Vertex :each device (Router, pc, etc..)
- Edges: connection between the devices.



## REAL TIME

- Transportation networks.

Vertices: In road networks vertices are intersections.
Edges : Road segments between intersection.

- Used by : Google maps, Bing maps and now Apple IOS 9 maps for map programs.


## VIBHAUG-1

IL SOCIETY
AKSHAR I SOCIE


Graph Theory
Presented by
Neeraj Anora

## Graph?

- Network = graph
- Informally a graph is a set of nodes joined by a set of lines or arrows called edges.



## Graph Theory History

- Leonhard Euler's paper on "Seven Bridges of Konigsberg"
- William R. Hamilton on "Cycles in Platonic graphs"
- Gustav Kirchhoff on "Trees in Electric Circuits"


## Graph Theory History (2)

- Arthur Cayley, George Polya on "Enumeration of Chemical Isomers"

- Auguste DeMorgan on "Four Colors of Maps"



## Exact Definition of Graph

- $G$ is an ordered triple $G:=(V, E, f)$ :
* $V$ is a set of nodes, points, or vertices.
* E is a set, whose elements are known as edges
\% $f$ is a function maps each element of $E$ to an unordered pair of vertices in V .
- Example:
$\nabla \mathrm{V}:=\{1,2,3,4,5,6\}$
- $E:=\{\{1,2\},\{1,5\},\{2,3\},\{2,5\},\{3,4\},\{4,5\},\{4,6\}\}$



## Type of Graph?



Simple graphs are graphs without multiple edges or self-loops.



Edges have directions: An edge is an ordered pair of nodes

Weighted Graph
is a graph for which each edge has an associated weight


## Connectivity and Graphs?

- a graph is connected if :
y you can get from any node to any other by following a sequence of edges
- any two nodes are connected by a path.
- A directed graph is strongly connected if there is a directed path from any node to any other node.

Degree:

- Number of edges incident on a node.
- Directed Graphs:
- In-degree: Number of edges entering
- Out-degree: Number of edges leaving
- Degree $=$ indeg + outdeg


Facts:

- If $G$ is a graph with $m$ edges, then:
$\wedge \Sigma \operatorname{deg}(\mathrm{v})=2 \mathrm{~m}=2|\mathrm{E}|$
- If $G$ is a directed graph then:
$\checkmark \Sigma$ indeg $(v)=\Sigma$ outdeg $(v)=|E|$
- Number of Odd degree Nodes is even


## Path:

- Walk: A walk of length $k$ in a graph is a succession of $k$ (not necessarily different) edges of the form.
- Ex: 1,2,5,2,3,4
- Path: A path is a walk in which all the edges and all the nodes are different
- Ex: 1,2,3,4,6
- Shortest Path also known geodesic path.
$\checkmark$ Diameter The longest shortest path in the graph.


## Cycle and Tree

- Cycle: A cycle is a closed path in which all the edges are different.
- Ex: 1,2,5,1 (3-Cycle)
- Tree: Connected Acyclic Graph and Two nodes have exactly one path between them



## Representing a graph:



Graph


|  | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0}$ | 0 | 210 | 0 | 450 | 0 |
| $\mathbf{1}$ | 210 | 0 | 60 | 190 | 0 |
| $\mathbf{2}$ | 0 | 60 | 0 | 130 | 200 |
| $\mathbf{3}$ | 450 | 190 | 130 | 0 | 0 |
| $\mathbf{4}$ | 0 | 0 | 200 | 0 | 0 |

Adjacency List

## References

- Daniel Bilar, "Some Graph Theory for Network Analysis", Lecture Note of Wellesley College.
- Keijo Ruohonen, "GRAPH THEORY"

Thanks


# Graph Theory Applications <br> BSC MSCS 3A <br> LIKITHA 01 <br> HARIKA 16 <br> KIRAN KUMAR 25 

## VERTEX COLOURINGS

k- Vertex colouring:
A k -vertex colouring of a graph G is an assignment of k colours, $1,2, \ldots, k$, to the vertices of G . The colouring is proper if no two distinct adjacent vertices have the same colour.
k- Vertex colorable:

A graph G is k -vertex colorable if G has a proper k -vertex colouring. k - vertex colorable is also called as k -colorable. A graph is k-colorable if and only if its underlying simple graph is k -colorable.

Uniquely k-colorable:
A graph $G$ is called uniquely k-colorable if any two proper k - colourings of G induce the same partition of V .

K-Chromatic:
If $\chi(G)=k, G$ is said to be $k$-chromatic.
Critical graph:
A graph $G$ is critical if $\chi(\mathrm{H})<\chi(\mathrm{G})$ for every proper sub graph $H$ of $G$. Such graphs were first investigated by Dirac (1952).
k-critical:
A k-critical graph is one that is k-chromatic and critical.Every k-chromatic graph has a k-critical subgraph.

## EDGE COLOURINGS

K -edge colouring:
A k-edge colouring of a loop less graph $G$ is an assignment of $k$ colours, $1,2, \ldots, k$, to the edges of $G$. The colouring is proper if no two distinct adjacent edges have the same colour.

K-edge colorable:
A graph G is k -edge colorable if G has a proper k-edge colouring. Edge chromatic number $\chi^{\prime}(\mathrm{G})$ :

The edge chromatic number $\chi^{\prime}(\mathrm{G})$,of a loop less graph $G$ is the minimum $k$ for which $G$ is $k$-edge colorable.

K-edge chromatic:
If $\chi^{\prime}(\mathrm{G})=\mathrm{k}, \mathrm{G}$ is said to be k -edge chromatic.
Let $\ell$ be a given k-edge colouring of G . We shall denote by $c(v)$ the number of distinct colors represented at $v$. Clearly, we always have $\mathrm{c}(\mathrm{v}) \leq \mathrm{d}(\mathrm{v}) \rightarrow(2)$. Moreover, $\ell$ is a proper $k$-edge colouring if and only if equality holds in (2) for all vertices in G.

## APPLICATIONS OF EDGE COLOURINGS

## The Timetabling Problem:

In aschool there are $m$ teachers $X_{1}, X_{2}, \ldots, X_{m}$ and $n$ dasses $\mathrm{Y}_{1}, \mathrm{Y}_{2}, \ldots, \mathrm{Y}_{\mathrm{n}}$. Given the teacher $\mathrm{X}_{\mathrm{i}}$ is required to teach class $\mathrm{Y}_{\mathrm{j}}$ for $\mathrm{p}_{\mathrm{ij}}$ periods, schedule a complete timetable in the minimum possible number of periods. This is known as the Timetabling Problem. We represent the teaching requirements by a bipartite graph G with bipartition $(X, Y)$, where $X=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ and $Y=\left\{y_{1}, y_{2}, \ldots\right.$, $\left.\mathrm{y}_{\mathrm{n}}\right\}$ and vertices $\mathrm{x}_{\mathrm{i}}$ and $\mathrm{y}_{\mathrm{j}}$ are joined by $\mathrm{p}_{\mathrm{ij}}$ edges. Now in any one period, each teacher can teach at most one class, and each class can be taught by at most one teacher. This is at least our assumption. Thus a teaching schedule for one period corresponds to a matching in the graph and, conversely, each matching corresponds to a possible assignmentof teachers toclasses for one period. Our problem, therefore is to partition the edges of G into as few matching as possible or, equivalently, to properly colour the edges of $G$ with as few colors as possible. Since G is bipartite, we know by theorem $1.35 \chi^{\prime}(\mathrm{G})=\Delta(\mathrm{G})$. Hence, if no teacher teaches for
more than p periods, the teaching requirements can be scheduled in a p-period timetable. We thus have a complete solution to the problem.

## Graph coloring techniques in scheduling:

Here some scheduling problems that uses variants of graph coloring methodologies such as precoloring, list Coloring, multicoloring, minimum sum coloring are given in brief.

## Job scheduling:

Here the jobs are assumed as the vertices of the graph and there is an edge between two jobs if they cannot be executed simultaneously. There is a 1-1 correspondence between the feasible scheduling of the jobs and the colorings of the graph. [4]

## Aircraft scheduling:

Assuming that there are k aircrafts and they have to be assigned n flights. The $\mathrm{i}^{\text {th }} \quad$ flight should be during the time interval $\left(a_{i}, b_{i}\right)$. If two flights overlap, then the same aircraft cannot be assigned to both the flights. This problem is modeled as a graph as follows.

The vertices of the graph correspond to the flights. Two vertices will be connected, if the corresponding time intervals overlap. Therefore, the graph is an interval graph that can be colored optimally in polynomial time. [4]

## Bi-processor tasks:

Assume that there is a set of processors and set of tasks. Each task has to be executed on two processors
simultaneously and these two processors must be pre assigned to the task. A processor cannot work on two jobs simultaneously. This type of tasks will arise when
scheduling of file transfers between processors or in case of mutual diagnostic besting of processors. This can be modeled by considering a graph whose vertices correspond to the processes and if there is any task that has to be executed on processors i and j , then and edge to be added between the two vertices. Now the scheduling problem is to assign colors to edges in such a way that every color appears at most o nce at a vertex.

If there are no multiple edges in the graph (i.e) no two tasks require the same two processors then the edge coloring technique can be adopted. The authors have developed an algorithm for multiple edges which gives an 1-1 approximate solution. [4]


Figure - 3 Tasks allocated to processors
The diagram shows the tasks namely task1, task2, task3 and task4 are allocated to the processors (P1, P5); (P1, P6); $(\mathrm{P} 2, \mathrm{P} 4)$ and $(\mathrm{P} 3, \mathrm{P} 7)$ respectively

## Minimum sum coloring:

In minimum sum coloring, the sum of the colors assigned to the vertices is minimal in the graph. The minimum sum coloring technique can be applied to the scheduling theory of minimizing the sum of completiontimes of the jobs. The multicolor version of the problem can be used to model jobs with arbitrary lengths. Here, the finish time of a vertex is the largest color assigned to it and the sum of coloring is the sum of the finish time of the vertices. That is the sum of the finish times in a multicoloring is equal to the sum of completion times in the corresponding schedule.[4]

## Time table scheduling:

Allocation of classes and subjects to the professors is one of the major issues if the constraints are complex.Graph theory plays an important role in this problem. For m professors with n subjects the available number of p periods timetable has to be prepared. This is done as follows.

Figure - 4

| p | $\mathrm{n}_{1}$ | $\mathrm{n}_{2}$ | $\mathrm{n}_{3}$ | $\mathrm{n}_{4}$ | $\mathrm{n}_{5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~m}_{1}$ | 2 | 0 | 1 | 1 | 0 |
| $\mathrm{~m}_{2}$ | 0 | 1 | 0 | 1 | 0 |
| $\mathrm{~m}_{3}$ | 0 | 1 | 1 | 1 | 0 |
| $\mathrm{~m}_{4}$ | 0 | 0 | 0 | 1 | 1 |

A bipartite graph (or bigraph is a graph whose vertices can be divided into two disjoint sets $U$ and $V$ such that every edge connects a vertex in $U$ to one in $V$; that is, $U$ and $V$ are independent sets[7]) G where the vertices are the number of professors say $m 1, m 2, m 3, m 4, \ldots \ldots . m_{k}$ and n number of subjects say $\mathrm{n}_{1}, \mathrm{n}_{2}, \mathrm{n}_{3}, \mathrm{n} 4, \ldots \ldots . \mathrm{n}_{\mathrm{m}}$ such that the vertices are connected by pi edges. It is presumed that at any one period each professor can teach at most one subject and that each subject can be taught by maximum one professor. Consider the first period. The timetable for this single period corresponds to a matching in the graph and conversely, each matching corresponds to a possible assignment of professors to subjects taught during that period. So, the solution for the timetabling problem will be obtained by partitioning the edges of graph G into minimum number of matching. Also the edges have to be colored with minimum number of colors. This problem can also be solved by vertex coloring algorithm. " The line graph $\mathrm{L}(\mathrm{G})$ of G has equal number of vertices and edges of $G$ and two vertices in $L(G)$ are connected by an edge iff the corresponding edges of G have a vertex in common. The line graph $\mathrm{L}(\mathrm{G})$ is a simple graph and a proper vertex coloring of $\mathrm{L}(\mathrm{G})$ gives a proper edge coloring of G by the same number of colors. So, the problem can be solved by finding minimum proper vertex coloring of $\mathrm{L}(\mathrm{G})$." For example, Consider there are 4 professors namely $m_{1}, m_{2}, m_{3}, m_{4}$, and 5 subjects say $\mathrm{n}_{1}, \mathrm{n}_{2}, \mathrm{n}_{3}, \mathrm{n}_{4}, \mathrm{n} 5$ to be taught. The teaching requirement matrix $p=\left[p_{i j}\right]$ is given below.

## Map coloring and GSM mobile phone networks:

Groups Special Mobile (GSM) is a mobile phone network where the geographical area of this network is divided into hexagonal regions or cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the neighbors. Since GSM operate only in four different frequency ranges, it is clear by the concept of graph theory that only four colors can be used to color the cellular regions. These four different colors are used for proper coloring of the regions. Therefore, the vertex coloring algorithm may be used to assign at most four different frequencies for any GSM mobile phone network.

### 3.1 Traveling Salesman Problem :

TSP is a very well-known problem which is based on Hamilton cycle. The problem statement is: Given a number of cities and the cost of traveling from any city to any other city, find the cheapest round-trip route that visits every city exactly once and return to the starting city. In graph terminology, where the vertices of the graph represent cities and the edges represent the cost of traveling between the connected cities (adjacent vertices), traveling salesman problem is just about trying to find the Hamilton cycle with the minimum weight. This problem has been shown to be NP-Hard. Even though the problem is computationally difficult, a large number of heuristics and exact methods are known, so that some instances with tens of thousands of cities have been solved. The most direct solution would be to try all permutations and see which one is cheapest (using brute force search). The running time for this approach is $O(V!)$, the factorial of the number of cities, so this solution becomes impractical even for only 20 cities. A dynamic programming solution solves the problem with a runtime complexity of $O(V 22 \mathrm{~V})$ by considering dp[end][state] which means the minimum cost to travel from start vertex to end vertex using the vertrices stated in the state (start vertex can be any vertex chosen at the start). As there are
$V 2 V$ subproblems and the time complexity to solve each sub-problems is $O(V)$, the overall runtime complexity $O(V 22 \mathrm{~V})$.

## SECTION 4.1: CHEMICAL APPLICATIONS

Recall that the eigenvalues of $L(G)$, the Laplacian matrix of graph $G$ are
$\lambda_{1} \geq \lambda_{2} \geq \ldots \geq \lambda_{\mathrm{n}}$ where $\lambda_{\mathrm{n}}=0$ and $\lambda_{\mathrm{n}-1}>0$ if and only if $G$ is connected. Also recall that a tree is a connected acyclic graph.

A chemical tree is a tree where no vertex has a degree higher than 4 . Chemical trees are molecular graphs representing constitutional isomers of alkanes. If there are $n$ vertices, each chemical tree represents a particular isomer of $\mathrm{C}_{\mathrm{n}} \mathrm{H}_{2 \mathrm{n}+2}$. The first four are methane, ethane, propane, and butane. After that, the alkanes are named based on Greek numbers. For example, $\mathrm{C}_{5} \mathrm{H}_{12}$ is pentane. Compounds whose carbons are all linked in a row, like the two below, are called straight-chain alkanes. For example, if $n=1$, we have the graph in Figure 4-1, which represents methane.


= Hydrogen
Figure 4-1 Methane

Figure 4-2 shows us butane, which is $\mathrm{C}_{4} \mathrm{H}_{10}$.


Figure 4-2 Butane

Compounds that have the same formula, but different structures, are called isomers. When $\mathrm{C}_{4} \mathrm{H}_{10}$ is restructured as in Figure 4-3, we have isobutane, or 2-

Methylpropane. Butane and 2-Methylpropane are isomers.


Figure 4-3 Isobutane or 2-Methylpropane

Compounds with four carbons have 2 isomers, while those with five carbons have 3 isomers. The growth, however, is not linear. The chart below compares the number of carbons with the number of isomers.

| Formula | Number of <br> Isomers |
| :---: | :---: |
| $\mathrm{C}_{6} \mathrm{H}_{14}$ | 5 |
| $\mathrm{C}_{7} \mathrm{H}_{16}$ | 9 |
| $\mathrm{C}_{8} \mathrm{H}_{88}$ | 18 |
| $\mathrm{C}_{9} \mathrm{H}_{20}$ | 35 |
| $\mathrm{C}_{10} \mathrm{H}_{22}$ | 75 |
| $\mathrm{C}_{15} \mathrm{H}_{32}$ | 4,347 |
| $\mathrm{C}_{20} \mathrm{H}_{42}$ | 366,319 |

[Mc, p76]
When a carbon has four carbons bonded to it, we have a quarternary carbon. An example is below in Figure 4-4, which is called a 2,2-Dimetylpropane. It is isomeric to Pentane.


Figure 4-4

For simplicities sake, we will just draw the carbon atoms from this point on, with the understanding that there are enough hydrogen atoms attached to each carbon to give that carbon atom a degree of 4 .

Study was done on the eigenvalues of molecular graphs, and in particular, $\lambda_{1}$, the largest eigenvalue of a graph. When the isomeric alkanes are ordered according to their $\lambda_{1}$ values, regularity is observed. [Gu, p 408]

Let $\Delta$ denote the maximum degree of a graph. The chemical trees that pertain to the 18 isomeric octanes $\mathrm{C}_{8} \mathrm{H}_{18}$ follow a pattern with respect to their largest eigenvalue, $\lambda_{1}$. The isomer with the smallest $\lambda_{1}(3.8478)$ value is the straight-chain octane in Figure $4-5$, that has $\Delta=2$.


## Figure 4-5

The next 10 isomers have various extensions of branching, but none possess a quaternary carbon atom. All of them have $\Delta=3$, and their $\lambda_{1}$ 's are greater than that of the straight-chain graph in Figure 4-5, where $\Delta=2$, and less than the following seven, who have $\Delta=4$. They are shown below in Figure 4-6.


08000
Figure 4-6




 090

The $12^{\text {th }}$ through the $18^{\text {th }}$ octanes contain a quaternary carbon atom, they all have $\Delta=4$, and they have the largest $\lambda_{1}$. The largest one has $\lambda_{1}=5.6458$ and is the last tree shown below in Figure 4-7.






Figure 4-7



This same regularity occurs with isomeric alkanes with $n$ carbon atoms, discussed above. The normal alkane with $\Delta=2$ has the smallest $\lambda_{1}$. All alkanes with $\Delta=3$ have $\lambda_{1}$ greater than the alkanes with $\Delta=2$, and smaller than any isomer with $\Delta=4$. We can therefore draw the conclusion that $\Delta$, which tells us whether or not there is a quaternary
carbon atom, is the main molecular structure descriptor affecting the value $\lambda_{1}$, the largest
Laplacian eigenvalue of an alkane. It has been discovered that $\lambda_{1}$ can be bounded by

$$
\Delta+1<\lambda_{1}<\Delta+1+2 \sqrt{\Delta-1}
$$

Also, by using a linear combination of the lower and upper bounds, $\lambda_{1}$ can be estimated by

$$
\lambda_{1} \approx \Delta+1+\gamma \sqrt{\Delta-1},
$$

where $\gamma$ depends on both $n$ and $\Delta$. For alkanes, it has been discovered through numerical testing that $\gamma \approx 0.2$. [Gu p 410]

It is possible to establish the alkane isomers with $\Delta=3$ or $\Delta=4$ that have the minimal $\lambda_{1}$. Give $\mathrm{P}_{\mathrm{n}}$, below, $\mathrm{T}_{n}{ }^{\min }$ is the tree that establishes the minimal $\lambda_{1}$ for $\Delta=3$, and $Q_{n}{ }^{\text {min }}$ is the tree that establishes the minimal $\lambda_{1}$ for $\Delta=4$.



Figure 4-8

The structure trees that represent the maximal $\lambda_{1}$ are more complex. The $\mathrm{T}_{n}{ }^{\max }$ and $\mathrm{Q}_{n}{ }^{\text {max }}$ coincide with the chemical trees that have the same $\Delta$ and $n$, having maximal $\lambda_{1}$ and minimal W , where W represents the Wiener topological index of alkanes, and conforms to the formula $\mathrm{W}=n \sum_{i=1}^{n-1} \frac{1}{\lambda_{i}}$. The exact characterizations of these trees are complex, and will not be covered here.

## Thank you

Our source: https://www.academia.edu


## Radical Plane

To show that the locus of points whose powers with respect to two spheres are equal is a plane perpendicular to the line joining their centres. The powers of the point $P(x, y, z)$ with respect to the spheres

$$
\begin{aligned}
& S_{1}=x^{2}+y^{2}+z^{2}+2 u_{1} x+2 v_{1} y+2 w_{1} z+d_{1}=0 \\
& S_{2}=x^{2}+y^{2}+z^{2}+2 u_{2} x+2 v_{2} y+2 w_{2} z+d_{2}=0
\end{aligned}
$$

are
and

$$
x^{2}+y^{2}+z^{2}+2 u_{1} x+2 v_{1} y+2 w_{1} z+d_{1}
$$

$$
x^{2}+y^{2}+z^{2}+2 u_{2} x+2 v_{2} y+2 w_{2} z+d_{2}
$$

respectively


Equating this we obtain
$2 \mathrm{x}\left(\mathrm{u}_{1}-\mathrm{u}_{2}\right)+2 \mathrm{y}\left(\mathrm{v}_{1}-\mathrm{v}_{2}\right)+2 \mathrm{z}\left(\mathrm{w}_{1}-\mathrm{w}_{2}\right)+\left(\mathrm{d}_{1}-\mathrm{d}_{2}\right)=0$
which is the required locus, and being of the first degree in ( $x, y, z$ ), it represents a plane which is obviously perpendicular to the line joining the centres of the two spheres and is called the radical plane of the two spheres.

Thus the radical plane of the two spheres
$S_{1}-S_{2}=0$
In case the two spheres intersect, the plane of their common circle is their radical plane.

## Radical line



The three radical planes of three spheres taken two by two intersect in a line.

If

$$
S_{1}=0, S_{2}=0, S_{3}=0
$$

be the three spheres, their radical planes

$$
S_{1}-S_{2}=0, S_{2}-S_{3}=0, S_{3}-S_{1}=0
$$

clearly meet in the line

$$
S_{1}=S_{2}=S_{3}
$$

This line is called the radical line of the three spheres.

## RADICAL CENTER

The four radical lines of four spheres taken three by three intersect at a point. The point common to the three planes

$$
S_{1}=S_{2}=S_{3}=S_{4}
$$

is clearly common to the radical lines, taken three by three, of the four spheres

$$
S_{1}=0, S_{2}=0, S_{3}=0, S_{4}=0 .
$$

This point is called the radical centre of the four spheres.

## THANK YOU

## GRAPH THEORY TREES AND ITS PROPERTIES

Tree (graph theory)


Section 1.4: Graphs and Trees
A graph is a set of objects (called vertices or nodes) and edges between pairs of nodes


Vertices $=\{\mathrm{Ve}, \mathrm{G}, \mathrm{S}, \mathrm{F}, \mathrm{Br}, \mathrm{Co}, \mathrm{Eq}, \mathrm{Pe}, \mathrm{Bo}, \mathrm{Pa}, \mathrm{Ch}, \mathrm{A}, \mathrm{U}\}$ Edges $=\{\{\mathrm{Ve}, \mathrm{G}\},\{\mathrm{Ve}, \mathrm{Br}\}, \ldots$

## TREES IN GRAPH THEORY

- In graph theory, a tree is an undirected graph in which any two vertices are connected by exactly one path, or equivalently a connected acyclic undirected graph.
- The term "tree" was coined in 1857 by the British mathematician Arthur Cayley.
- Ex-: The undirected graph representing a family tree(restricted to people of just one gender is an example of a tre.

A tree is an undirected graph $G$ that satisfies any of the following equivalent conditions:

- $G$ is connected and acyclic (contains no cycles).
- $G$ is acyclic, and a simple cycle is formed if any edge is added to $G$.
- $G$ is connected, but would become disconnected if any single edge is removed from $G$.
- $G$ is connected and the 3 -vertex complete graph $K_{3}$ is not a minor of $G$.
- Any two vertices in $G$ can be connected by a unique simple path.

| A labeled tree with 6 <br> vertices and 5 edges. |  |
| :--- | :--- |
| Vertices | $v$ |
| Edges | $v-1$ |
| $\frac{\text { Chromatic }}{}$ | 2 if $v>1$ |
| number |  |
| Table of graphs and |  |



## PROPERTIES OF TREES

- A tree with $n$ vertices has $n$-ledges.
-A full $m$-ary tree with $i$ internal vertices contains $n=m i+\mid$ vertices.
- A full m-ary tree with
i) n vertices has $\mathrm{i}=(\mathrm{n}-\mathrm{I}) / \mathrm{m}$ internal vertices and $\mathrm{I}=[(\mathrm{m}-\mathrm{I}) \mathrm{n}+\mathrm{I}] / \mathrm{m}$ leaves,
ii) internal vertices has $n=m i+\mid$ vertices and $I=(m-I) i+\mid$ leaves, iii)! leaves has $\mathrm{n}=(\mathrm{ml}-\mathrm{I}) /(\mathrm{m}-\mathrm{I})$ vertices and $\mathrm{i}=(\mathrm{L}-\mathrm{I}) /(\mathrm{m}-\mathrm{I})$ internal vertices.


## APPLICATIONS OF TREES

## - I)BINARY TREES

Searching for items in a list is accomplished in computer science is accomplished using binary trees.

- 2)DECISION TREES

A rooted tree in which each internal vertex corresponds to a decision, with a sub tree at these vertices for each possible outcome of the decision is called a decision.

## - 3)PREFIX CODES

Each character is encoded with a (potentially) different number of bits. ... This property defines a prefix code, and it allows us to represent the character encodings with a binary tree, as shown above. To decode a given bit string: Start at the root of the tree.

## THANK YOU

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## ASSIGNMENT GRAPH THEORY

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## DEFINITIONS :

- An edge of a graph joins a node to itself is called a loop or self-loop.
- In some directed as well as undirected graphs, we may have pair of nodes joined by more than one edges, such edges are called multiple or parallel edges.
- A graph which has neither loops nor multiple edges i.e. where each edge connects two distinct vertices and no two edges connects the same pair of vertices is called a simple graph.
- Any graph which contains some multiple edges is called a multigraph. In a multigraph, no loops are allowed.


## TYPES OF GRAPH:



## APPLICATIONS:

- GPS or Google Maps:

GPS or Google Maps are to find a shortest route from one destination to another. The destinations are Vertices and their connections are Edges consisting distance. The optimal route is determined by the software. Schools/ Colleges are also using this technique to pick up students from their stop to school. Each stop is a vertex and the route is an edge. A Hamiltonian path represents the efficiency of including every vertex in the route.


- Social Networks:

We connect with friends via social media or a video gets viral, here user is a Vertex and other connected users create an edge therefore videos get viral when reached to certain connections.


- Traffic lights:

The functioning of traffic lights i.e. turning Green/Red and timing between them. Here vertex coloring technique is utilized to solve conflicts of time and space by identifying the chromatic number for the number of cycles needed.


- To clear road blockage:

When roads of a city are blocked due to ice. Planning is needed to put salt on the roads. Then Euler paths or circuits are used to traverse the streets in the most efficient way.


- GSM Mobile Phone Networks and Map Coloring:

All mobile phones connect to the GSM network by searching for cells in the neighbors. Since GSM operate only in four distinct frequency ranges, it is clear by the concept of graph theory that only four colors may be utilized to color the cellular regions. These four different colors are used for proper coloring of the regions. The vertex coloring algorithm can be used to allocate at most four distinct frequencies for any GSM mobile phone network.

- Graphs in OR:

Graph theory is dynamic tool in combinatorial operations research. Some important Operation Research problems which can be explained using graphs are given here. Transport network is used to model the transportation of commodity from one destination to another destination. The objective is to maximize the flow or minimize the cost within the suggested flow. The graph theory is established as more competent for these types of problems though they have more constraints.

THANK YOU

## PLANE SECTION OF THE SPHERE

Roll no.: 107219474011-20

Definition

Consider a plane and a Sphere, we suppose that sphere and plane have points in common i.e. the interception of sphere and plane at these set of common points is called Plane section of a Sphere.



## EQUATION OF PLANE SECTION OF SPHERE

The equation of the
sphere withorigin as
centreand radi us ris
given by

$$
x^{2}+y^{2}+z^{2}=r^{2}
$$

$\cdots \cdots$ (1)
Let C (a,b, c) be the
centreof theplane
section of the sphere

whose equation we
have tofindout. The
linesegment OC dr a wn

Let $P(x, y, z) b e a n y p o i n t o n t h e$ plane section.

The direction ratio of PC are $x$ $a, y-b, z-c$ and it is perpendicular too.

Using the condition of perpendicularity, we have

$$
(x-a) a+(y-b) b+(z-c) c=0 \cdots(1 i)
$$

Equation (i) is satisfied by the coordinates of any point $P$ on $n$ he



Great Gircle

## 

- DEFIUTIOH OF GBEAT CIBCLE: A GRFAT CIRCLFOF A SPIERE IS TAE MIEBEXGTOH OF IEESFIERE AMD A PLANE WATGA PASSES THROUGH THE CENTER POINT OF THE SPHERE.


## Small Circle

SMALL CIRCLE OF A SPHERE IS DEFIMED AS THE INTERSECTION OF A SPHERE AND A PLANE, IF THE PLANE DOES NOT PASS THROUGH THE CENTER OF THE SPHERE


Thank You


## Warm Up

Determine whether the two polygons are similar. If so, give the similarity ratio.
1)

2)


1) Not similar.
2) Similar. Similarity ratio $=1.7$

## Solid Geometry



Three-dimensional figures, or solids, can be made up of flat or curved surfaces. Each flat surface is called a face. An edge is the segment that is the intersection of two faces. A vertex is the point that is the intersection of three or more faces.

## Three-Dimensional Figures

TERM
Prism is formed by two parallel congruent
olygonal faces called bases connected by
faces that are parallelograms.

A cylinder is formed by two parallel congruent circular bases and curved surface that connects the bases.


A pyramid is formed by a polygonal base and triangular faces that meet at a common vertex.

A cone is formed by a circular base and a curved surface that connects the base to a vertex.

A cube is a prism with six square faces. Other prisms and pyramids are named for the shape of their bases.



Triangular pyramid

$\square$
Hexagonal pyramid


By B.Sc(MPCs 3) Roll No. 10721946731-40

## What is circle ?

- Circle is the locus of points equidistant from a given point, the center of the circle. The common distance from the center of the circle to its points is called radius. Thus a circle is completely defined by its center $(O)$ and radius (R):

- $C(O, R)=O(R)=\{x: \operatorname{dist}(O, x)=R\}$.

- $O$ is the center of the circle.
- The distance between any of the points and the centre is called the radius. It can also be defined as the locus of a point equidistant from a fixed point.
- The diameter of a circle is the length of the line through the center and touching two points on its edge.
- C is the circumference. Circumference is the linear distance around the edge of a closed curve or circular object.

- Chord is a line segment whose endpoints lie on the circle
- Secant is an extended chord, a coplanar straight line cutting the circle at two points.
- Tangent is a coplanar straight line that touches the circle at a single point.
- Arc is any connected part of the circle.
- Circular sector a region bounded by two radius and an arc lying between the radius.
- Circular segment is a region, not containing the centre, bounded by a chord and an arc lying between the chord's endpoints.


## Area of sector

To calculate the area of sector first we need to know the center angle

$$
\text { area }=\pi r^{2}\left(\frac{C}{360}\right)
$$

where:
$C$ is the central angle in degrees
$r$ is the radius of the circle of which the sector is part.
$\pi$ is Pi, approximately 3.142

If only the arc length is given

$$
\text { area }=\frac{1}{2} R I
$$

where:
$L$ is the arc length.
$R$ is the radius of the circle of which the sector is part.

$A=\left(\frac{a}{360}\right)+m r^{2}$

## Area of segment

- The Area of a Segment is the area of a sector minus the triangular piece (shown in light blue here).
- There is a lengthy reason, but the result is a slight modification of the Sector formula:
- Area of Segment $=1 / 2 \times(\boldsymbol{\vartheta}-\sin \boldsymbol{\vartheta}) \times r^{2} \quad($ when $\boldsymbol{\vartheta}$ is in radians)



## Formula for calculating a circle

- The diameter of a circle is twice of the radius diameter $=2 R$ where: $R$ is the radius of the circle
- The circumference is related to the radius and diameter by

$$
C=2 \pi r=\pi d
$$

- Area of a circle is given by the formula

Area $=\pi r^{2}$.

## Sphere

- Sphere can be define as a three dimensional closed body with all points on its surface at an equal distance from a single central point.
- A sphere is defined mathematically as the set of points that are all the same distance r from a given point in three-dimensional space. This distance $r$ is the radius of the sphere, and the given point is the center of the sphere. The maximum straight distance through the sphere passes through the center and is thus twice the radius; it is the diameter.


## Calculation of sphere

- The surface are of a sphere is

$$
A=4 \pi r^{2}
$$

- For the enclosed volume inside a sphere is derived to be

$$
V=\frac{4}{3} \pi r^{3}
$$

## Circle VS sphere

- Similarity - both circles and spheres is that both have a perfect symmetry around their centers. All the points lying at a distance rfrom the center of the sphere or a circle form a sphere. The longest distance inside a sphere is double this distance $r$ and is called the diameter of the sphere. To a mathematician, both the circle and a sphere are one and the same thing as a collection of all the points that are equidistant ${ }^{\text {® }}$ from the center of the circle or the sphere. In a plane a round object is called a circle but the same circle becomes a sphere in space.
- Differences - circle is a figure, a sphere is an object. A circle is a 2D figure whereas a sphere is a 3D object having volume. One can only calculate the surface area of a circle whereas it is possible to calculate the volume of a sphere.



# Bhavan's Vivekananda College 

of Science, Humanities and Commerce
Sainikpuri, Secunderabad-500 094
(Accredited with 'A' Grade by NAAC)
Autonomous College - Affiliated to Osmania University
Report for the year 2019-20

1. Name of the Department if the programme is conducted inside the college: DEPARTMENT OF MATHEMATICS AND STATISTICS
2. Date of the event: DEC. 10,2019
3. Place of the event: Room No. 71.bHAVAN's VIVEKANANDA COLLEGE
4. Name(s)of the Resource person(s):

Dr.Aparna, Sr.Assistant professor in VNRVJIET
5. Description of the Event: (within 500 words with 3 relevant pictures) Dr.Aparna, Sr.Assistant professor in VNRVJIET gave a talk on 'vectors and it's applications'.The talk was for students of B.Sc III year on December $10^{\text {th }} 2019$ in Room No. 71.The students had a paper on Vector Calculus in their sem 5.This talk was on the applications. The speaker give an insight into the physical interpretations of different concepts of which the students learnt only the mathematical aspects.
In the fifth semester students had various concepts like gradient divergence, curl,irrotational, solenoidal vector.
Divergence:
Consider water flowing through a large pipe. Now, it has smaller pipes joined to it. Hence, as the water flows, more water is added along the way by the smaller pipes. Hence, the mass flow rate increases as the water flows.


In another case, consider that there is a leakage in the pipe. Hence the mass flow rate decreases as it flows. This change in the flow rate through the pipe, whether it increases or decreases, is called as divergence. Divergence denotes only the magnitude of change and so, it is a scalar quantity. It does not have a direction.

When the initial flow rate is less than the final flow rate, divergence is positive (divergence > 0 ). If the two quantities are same, divergence is zero. If the initial flow rate is greater than the final flow rate divergence is negative (divergence $<0$ ).

Curl:
Imagine pouring water in a cup. The water won't just low linearly but rather, as it reaches the end of the cup, it will flow in a rotational motion before settling in the cup. Or consider water draining down the sink, it will swirl in a rotational motion before going out. If we plot this rotational flow of water as vectors and measure it, it will denote the Curl.

Curl is a measure of how much a vector field circulates or rotates about a given point. when the flow is counter-clockwise, curl is considered to be positive and when it is clock-wise, curl is negative. Sometimes, curl isn't necessarily flow around a single time. It can also be any rotational or curled vector.

Learning about gradient, divergence and curl are important They help us calculate the flow of liquids and correct the disadvantages. For example, curl can help us predict the voracity, which is one of the causes of increased drag. By using curl, we can calculate how intense it is and reduce it effectively. Calculating divergence helps us understand the flow rate and correct it to suit our needs.

In vector calculus a solenoidal vector field (also known as an incompressible vector field, a divergence-free vector field, or a transverse vector field) is a vector
field v with divergence zero at all points in the field:


The speaker has given a clear picture of the physical interpretation in fluid dynamics.The concept of energy flow and heat transfer in a machine in vehicle was explained clearly.
The students were very happy with the content and relevance of the lecture.

